North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

3rd Grade Mathematics ● Unpacked Contents
For the new Standard Course of Study that will be effective in all North Carolina schools in the 2017-18 School Year.

This document is designed to help North Carolina educators teach the 3rd Grade Mathematics Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.

What is the purpose of this document?
The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

What is in the document?
This document includes a detailed clarification of each standard in the grade level along with a *sample* of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

How do I send Feedback?
Link for: Feedback for NC's Math Unpacking Documents We will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
Link for: NC Mathematics Standards

North Carolina Course of Study – 3rd Grade Standards				
Standards for Mathematical Practice				
Operations & Algebraic Thinking	**Number & Operations in Base Ten**	**Number & Operations-Fractions**	**Measurement & Data**	**Geometry**
Represent and solve problems involving multiplication and division. NC.3.OA.1 NC.3.OA.2 NC.3.OA.3 *Understand properties of multiplication and the relationship between multiplication and division.* NC.3.OA.6 *Multiply and divide within 100.* NC.3.OA.7 *Solve two-step problems.* NC.3.OA.8 *Explore patterns of numbers.* NC.3.OA.9	*Use place value to add and subtract.* NC.3.NBT.2 *Generalize place value understanding for multi-digit numbers.* NC.3.NBT.3	*Understand fractions as numbers.* NC.3.NF.1 NC.3.NF.2 NC.3.NF.3 NC.3.NF.4	*Solve problems involving measurement.* NC.3.MD.1 NC.3.MD.2 *Represent and interpret data.* NC.3.MD.3 *Understand the concept of area.* NC.3.MD.5 NC.3.MD.7 *Understand the concept of perimeter.* NC.3.MD.8	*Reason with shapes and their attributes.* NC.3.G.1

Standards for Mathematical Practice

Practice	Explanation and Example
1. Make sense of problems and persevere in solving them.	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third grade students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.
2. Reason abstractly and quantitatively.	Mathematically proficient third grade students should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3. Construct viable arguments and critique the reasoning of others.	In third grade, mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students require extensive opportunities to generate various mathematical representations and to both equations and story problems, and explain connections between representations as well as between representations and equations. Students should be able to use all of these representations as needed. They should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. Use appropriate tools strategically.	Mathematically proficient third grader students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
6. Attend to precision.	Mathematically proficient third grader students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
7. Look for and make use of structure.	In third grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
8. Look for and express regularity in repeated reasoning.	Mathematically proficient students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Return to Standards

Represent and solve problems involving multiplication and division.

NC.3.OA.1 For products of whole numbers with two factors up to and including 10:

- Interpret the factors as representing the number of equal groups and the number of objects in each group.
- Illustrate and explain strategies including arrays, repeated addition, decomposing a factor, and applying the commutative and associative properties.

Clarification

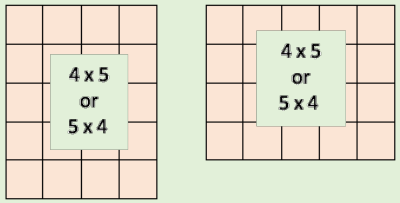
In this standard, students develop an understanding of multiplication of whole numbers. Students recognize multiplication as a means to determine the total number of objects (product) when there are a specific number of groups (factor) with the same number of objects in each group (factor). Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

While this standard focuses on strategies and representations of multiplication students should be given opportunities to explore and develop an understanding of multiplication by solving tasks and problems that are embedded in real-world contexts, which is the focus of NC.3.OA.3. Students should be working on NC.3.OA.3 while working on NC.3.OA.1.

Students build on their work with repeated addition and rectangular arrays from second grade. They also begin applying properties of multiplication.

The commutative property (order property) states that the order of numbers does not matter when you are adding or multiplying numbers.

For example: If a student knows that 5 x 4 = 20, then they also know that 4 x 5 = 20. There is no "fixed" way to write the dimensions of an array as rows x columns or columns x rows. Students should have flexibility in being able to describe both dimensions of an array.

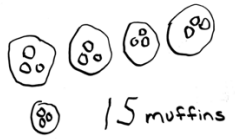


Students are introduced to the distributive property of multiplication, through decomposing a factor, as a strategy for solving multiplication problems. When finding the product of 9 x 7 a student may decompose 7 into 5 and 2 and rewrite the equation as 9 x 5 + 9 x 2. In Grade 3, parentheses may be used a grouping symbols but students should not be assessed on the usage of parentheses in the context of order of operations.

Checking for Understanding

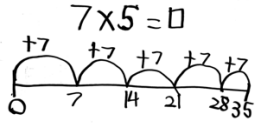
Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?

Possible response:

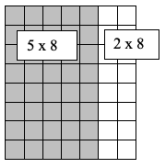


Sonya earns $7 a week pulling weeds. After 5 weeks of work, how much has Sonya worked? Write an equation and find the answer.

Possible response:



Joe has seven boxes of markers and each box has eight markers. Draw an array to determine how many markers Joe has by decomposing one of the factors. Then write an equation to match your picture.



Possible response:

My equation that matches my picture is 5x8 + 2x8.

Yasif is trying to find the product of 9 and 7. For each statement explain whether each strategy is an appropriate way to find the product.

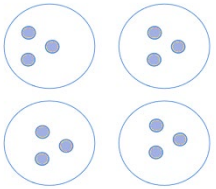
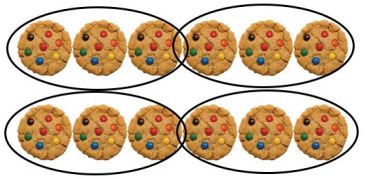
a. Draw an array with 9 rows and 7 columns and count the boxes
b. 5 x 5 + 4 x 2 = __
c. Jump 9 spots 7 times on a number line and ends at 63
d. Draw 9 circles and put the numbers 5 and 2 in each circle. Skip count first by 5s then by 2s. Add the ending numbers of each skip count together.

Return to Standards

Represent and solve problems involving multiplication and division.

NC.3.OA.2 For whole-number quotients of whole numbers with a one-digit divisor and a one-digit quotient:

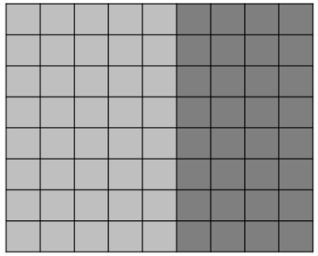
- Interpret the divisor and quotient in a division equation as representing the number of equal groups and the number of objects in each group.
- Illustrate and explain strategies including arrays, repeated addition, or subtraction, and decomposing a factor.

Clarification	**Checking for Understanding**
This standard focuses on two distinct models of division: partition models (fair share) and measurement (repeated subtraction) models. Partition models provide students with a total number and the number of groups. These models focus on the question, "How many objects are in each group so that the groups are equal?" Measurement (repeated subtraction) models provide students with a total number and the number of objects in each group. These models focus on the question, "How many equal groups can you make?" While this standard focuses on strategies and representations of division, students should be given opportunities to explore and develop an understanding of division by solving tasks and problems that are embedded in real-world contexts, which is the focus of NC.3.OA.3. Students should be working on NC.3.OA.3 while working on NC.3.OA.2	<u>Partition model:</u> There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag? *Possible response:* *Student A* *I skip counted by 4. 4, 8, 12.* *Since I skip counted 3 times that means that I need 3 cookies in each bag.*  *Student B* *I drew 4 bags. I then put 1 cookie in each bag until I had placed 12 cookies. I have 3 cookies in each bag.* <u>Measurement model:</u> There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill? *Possible Responses:* *Student A* *I drew groups of 3 cookies and circled them. I made 4 groups. Each group is a bag so I will fill 4 bags.*  *Student B* *I used repeated subtraction* *12 - 3 = 9* *9 - 3 = 6* *6 - 3 = 3* *3- 3 = 0* *I subtracted 3 4 times so there are 4 bags of cookies.* Describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Represent and solve problems involving multiplication and division.

NC.3.OA.2 For whole-number quotients of whole numbers with a one-digit divisor and a one-digit quotient:

- Interpret the divisor and quotient in a division equation as representing the number of equal groups and the number of objects in each group.
- Illustrate and explain strategies including arrays, repeated addition, or subtraction, and decomposing a factor.

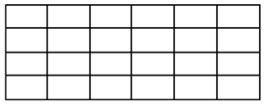
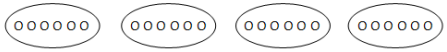
Clarification	Checking for Understanding
	Possible Responses: *There are 56 children playing soccer this year. They are placed on 8 teams with the same number of children on each team. How many children are on each team?* *The store has 56 pop-its in the bin. There are 8 pop-its per pack. How many packs of pop-its are there?* **Decomposing a factor** There are 72 tiles on the floor arranged in the shape as a rectangle. The patio looks like it has been divided into 2 smaller rectangles. The rectangle on the left has 40 tiles arranged in rows of 5. The rectangle on the right includes the rest of the tiles and includes the same number of rows. • Draw what the large rectangle looks like. • What equation can be used to show the dimensions and area of the rectangle on the left? • What equation can be used to show the dimensions and area of the rectangle on the right? • What equation can be used to show the dimensions and area of the large rectangle? *Possible Response:* *The rectangle on the left includes 40 tiles in rows of 5. 40 divided by 5 is 8 so there are 8 rows so the left rectangle equation is 8x5 = 40.* *The rectangle on the right has the rest of the tiles. 72 - 40 = 32.* *It has 32 tiles and I know it has 8 rows. I put 1 in each row until I added up to 32.* *8 x 1 = 8* *8 x 2 = 16* *8 x 3 = 24* *8 x 4 = 32* *There are 4 tiles in each row so the right rectangle is 8 x 4.* *The entire rectangle is 8x4 + 8 x 5 which is also 8 x 9 = 72.* 

Return to Standards

Represent and solve problems involving multiplication and division.

NC.3.OA.3 Represent, interpret, and solve one-step problems involving multiplication and division.

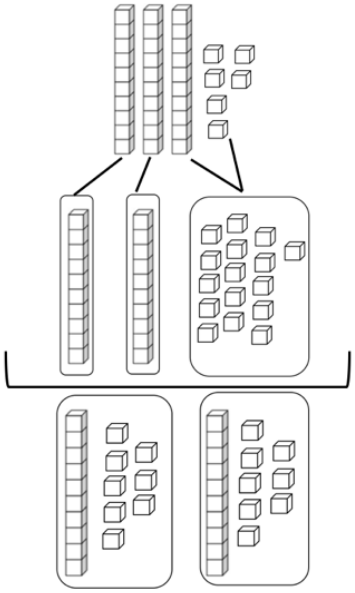
- Solve multiplication word problems with factors up to and including 10. Represent the problem using arrays, pictures, and/or equations with a symbol for the unknown number to represent the problem.
- Solve division word problems with a divisor and quotient up to and including 10. Represent the problem using arrays, pictures, repeated subtraction and/or equations with a symbol for the unknown number to represent the problem.

Clarification	**Checking for Understanding**
In this standard, students apply strategies from NC.3.OA.1 and NC.3.OA.2 to solve word problems that involve multiplication and division situations. This standard should be integrated into the work of NC.3.OA.1 and NC.3.OA.2 since research is clear that students more easily make sense of operations when math is embedded in real-world contexts (Carpenter et al., 2014; van de Walle et at., 2019). Students are expected to make connections between the various representations and strategies for multiplication and division, equations, and word problems. Students are expected to identify and create a word problem when given a specific equation, e.g., *Write a word problem that matches 32 ÷ 4 = __ .* The Multiplication & Division Situation table describes all the problem-solving situations that students are expected to solve independently by the end of the year. Students should be given ample experiences to explore all the different problem structures. Students are also expected to write equations that match word problems using unknowns. In previous grade levels students used a variety of pictures, such as stars, boxes, flowers to represent unknown numbers. In Grade 3, letters are also introduced as symbols to represent unknowns.	<u>Multiplication:</u> Each child has 3 t-shirts in their camp bag. There are 9 children. Write an equation using the letter S to represent the total number of shirts. Show your work to find the total number of shirts. *Possible responses:* *Student A* *My equation is 3 x 9 = S.* *I skip counted by 3s. 3, 6, 9, 12, 15, 18, 21, 24, 27. The 9th multiple is 27 so there are 27 shirts.* *Student B* *My equation is 9 x 3 = S.* *I broke the 9 into 5 and 4 since I know my 5s.* *I know 3 x 5 is 15. I then found 3 x 4 = 12. Since 15 + 12 = 27 there are 27 shirts.* *Student C* *My equation is 3 x 9 = S.* *I drew 9 circles and put the number 3 in each circle. I know that 3 x 10 = 30 so 3 x 9 is 3 less so 30 - 3 = 27. There are 27 shirts.* <u>Measurement model of division: unknown number of groups</u> There are 24 desks in the classroom. If the teacher puts 6 desks in each row. Write and equation that matches the problem that includes the letter D as an unknown for the number of rows of desks. How many rows are there? *Possible responses:*  *Student A* *My equation is 24 ÷ 6 = D.* *Student B* *My equation is 24 ÷ 6 = D.* *I drew 1 group of 6. I kept doing that until I had 24 circles. Since I have 4 groups of 6 there are 4 rows of chairs.* 

Represent and solve problems involving multiplication and division.

NC.3.OA.3 Represent, interpret, and solve one-step problems involving multiplication and division.

- Solve multiplication word problems with factors up to and including 10. Represent the problem using arrays, pictures, and/or equations with a symbol for the unknown number to represent the problem.
- Solve division word problems with a divisor and quotient up to and including 10. Represent the problem using arrays, pictures, repeated subtraction and/or equations with a symbol for the unknown number to represent the problem.

Clarification	Checking for Understanding
	Partition model of division: where the size of the groups is unknown: The bag has 36 hair clips, and Laura and her friend want to share them equally. How many hair clips will each person receive?  *36 hair clips are represented with base ten blocks* *Each girl receives 1 ten when 2 tens are divided evenly among them. There are 1 ten and 6 ones left. The ten is decomposed into ten ones.* *Each girl receives 8 ones along with the 1 ten.* *Each girl receives 18 hair clips.* Measurement model of division: where the number of groups is unknown Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	24 – 4 = 20	20 – 4 = 16	16 – 4 = 12	12 – 4 = 8	8 – 4 = 4	4 – 4 = 0

The bananas will last for 6 days.

Represent and solve problems involving multiplication and division.

NC.3.OA.3 Represent, interpret, and solve one-step problems involving multiplication and division.

- Solve multiplication word problems with factors up to and including 10. Represent the problem using arrays, pictures, and/or equations with a symbol for the unknown number to represent the problem.
- Solve division word problems with a divisor and quotient up to and including 10. Represent the problem using arrays, pictures, repeated subtraction and/or equations with a symbol for the unknown number to represent the problem.

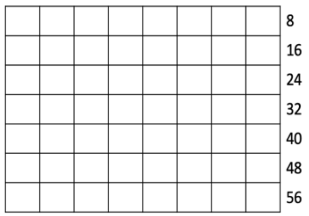
Clarification | **Checking for Understanding**

Multiplication & Division Situations

	Unknown Product **3 x 6 = ?**	**Group Size Unknown (How many in each group?" Division)** **3 x ? = 18 and 18 ÷ 3 = ?**	**Number of Groups Unknown ("How many groups?" Division)** **? x 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays & Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?

Return to Standards

Understand properties of multiplication and the relationship between multiplication and division.
NC.3.OA.6 Solve an unknown-factor problem, by using division strategies and/or changing it to a multiplication problem.

Clarification	Checking for Understanding
This standard calls for students to represent and solve unknown factor problems by applying the inverse relationship between multiplication and division. In earlier grades students applied their understanding of the equal sign that the sign means "the same amount as" to interpret an equation with an unknown. When given 4 x P = 40, they might think about that equation using reasoning such as this: • 4 groups of some number is the same as 40 • That means that 4 times some number is the same as 40 • Since I know that 4 groups of 10 is 40 so the unknown number is 10. The missing factor is 10 because 4 times 10 equals 40. Students are expected to use the multiplication and division strategies that are described in 3.OA.1 and 3.OA.2 while working with this standard.	<u>Reasoning</u> Sarah did not know the answer to 63 divided by 7. Is each of the following an appropriate way for Sarah to think about the problem? Explain why or why not with a picture or words for each one. • I know that 7 x 9 = 63, so 63 divided by 7 must be 9. • I know that 7x 10 = 70. If I take away a group of 7, that means that I have 7 x 9 = 63. So, 63 divided by 7 is 9. • I know that 63 divided by 7 can be solved using repeated subtraction. 63 - 7 - 7 - 7 - 7 - 7 - 7 - 7 - 7 - 7 = 63." The answer is 9 since I had to subtract 9 7's. • I know that 63 divided by 7 is like skip counting by 7s until I get to 63. I know that 7 x 5 = 35. Then I added on groups of 5 until I got to 65. My answer is 13 since 13 5's get me to 63. • I know that 7 x 5 is 35. 63 minus 35 is 28. I know that 7 x 4 = 28. So, if I add 7 x 5 and 7 x 4 I get 63. That means that 7 x 9 is 63, or 63 divided by 7 is 9. *Answer:* The first, second, third and fifth statements are appropriate. <u>Using area as a context</u> A rectangle has an area of 56 square feet and a length that is 8 feet long. Write a multiplication equation with the area, the length, and the unknown width. Show your work and find the width of the rectangle. *Possible response:* *Student A* *The multiplication equation is 8 x W = 56* *I found the width by skip counting from 40.* *I knew that 8 x 5 = 40 so I just added 8s. 8 x 6 = 48 and 8 x 7 = 56. The width is 7 feet.*  *Student B* *The multiplication equation is 8 x W = 56* *I found the width by drawing an array with 8 squares in each row.* *I kept adding rows until I had 56 squares.* *It took 7 rows, so the width is 7.*

Return to Standards

Multiply and divide within 100.

NC.3.OA.7 Demonstrate fluency with multiplication and division with factors, quotients, and divisors up to and including 10.

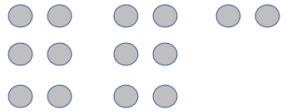
- Know from memory all products with factors up to and including 10.
- Illustrate and explain using the relationship between multiplication and division.
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Clarification	Checking for Understanding
This standard calls for students to be fluent with multiplication and division, which means they can demonstrate accuracy, efficiency, and flexibility with all combinations. Accuracy means they have the correct answer. Efficiency means that they can recall facts within 5 seconds (van de Walle, 2019). Flexibility means that they are able to mentally able to think about different ways to get the answer (e.g., 6 x 7 is the same as 6 x 5 plus 2 more groups of 6). Students develop fluency and begin to "know from memory" the multiplication and division combinations through ample experiences creating representations and applying the strategies that are described in NC.3.OA.1 and NC.3.OA.2 as well as conversations about the relationships and connections between factors and products as well as between divisors, dividends, and quotients. Specifically, students should explore that when you double one factor you also double the product. For example, if a student knows that 2 x 7 = 14 they can use the idea of doubling in order to consider the idea that 4 x 7 is the same as doubling 14. This idea of doubling allows students to make sense of combinations of 4s and 8s building off of their understanding of 2s, and also allows students to make combinations of 6s building off of their understanding of 3s. Please note that traditional flash cards and/or timed tests have not been proven as effective instructional strategies for developing fluency (Kling & Bay-Williams, 2014). Strategies students may use to attain fluency include: • Multiplication by zeros and ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s) • 3s facts, Doubling 3 facts (6s) • Tens facts (relating to place value, 5 x 10 is 5 tens or 50) • Five facts (half of tens) • Skip counting (counting groups of __ and knowing how many groups have been counted) • Square numbers (ex: 3 x 3) • Nines (10 minus 1 for each group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3) • Decomposing a factor (6 x 7 is 6 x 6 plus one more group of 6) • Commutative Property of Multiplication	<u>Activities to develop fluency (similar to NC.3.OA.1 and NC.3.OA.2)</u> Oak Elementary has 24 third graders. They are taking a field trip to a museum and want to have students in equal groups during the tour. Groups cannot be smaller than 2 students and not larger than 8 students. What size groups could they make? • Use your tiles or grid paper to show a model of how they could make the groups. • Draw a picture of your solutions. For each solution, write an equation. • Write a sentence to explain how you solved the problem. *Answers:* *The number of groups and group size should multiply to get 24 using numbers between 2 and 8. 3 groups of 8, 4 groups of 6, 6 groups of 4, 8 groups of 3.* Mr. Nala's class is making a garden. They bought 40 tomato plants. They want them in rows that have the same number of plants. There needs to be between 2 and 10 plants in each row. • Use your tiles to show a model of how they could make the garden. For each solution, write an equation. • Write a sentence to explain how you solved the problem. *Answers:* *The number of rows and the number of plants in each row should multiply to equal 40 and one of the numbers must be greater than 1 and less than 12.* *2 in each row and 20 rows, 4 in each row and 10 rows, 5 in each row and 8 rows, 8 in each row and 5 rows, 10 in each row and 4 rows.* <u>Illustrate and explain the relationship between multiplication and division</u> Bob knows that 2 × 9 = 18. How can he use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning.

Multiply and divide within 100.

NC.3.OA.7 Demonstrate fluency with multiplication and division with factors, quotients, and divisors up to and including 10.

- Know from memory all products with factors up to and including 10.
- Illustrate and explain using the relationship between multiplication and division.
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Clarification	Checking for Understanding
• Considering division as a missing factor problem (NC.3.OA.6), e.g. 24 divided by 4 is the same as what number times 4 equals 24. The last 2 bullet points talk about the inverse relationship between multiplication and division which is also addressed in NC.3.OA.6. The focus of this standard extends beyond the traditional notion of *fact families*, by having students explore the inverse relationship of multiplication and division. While exploring the unknown number in multiplication and division equations students should have exposure to multiplication and division problems presented in both vertical and horizontal forms. Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.	<u>Determine the unknown whole number in a multiplication or division equation relating three whole numbers</u> There are 14 children in the gym. Each child needs a partner for the three-legged race. How many pairs of children will there be? Write a division equation using P to represent the number of pairs. Write a multiplication equation using the same numbers and P to represent the number of pairs. Show your work and find the number of pairs that there will be. *Possible answers:* *Student A* *My division equation is 14 ÷ 2 = P.* *My multiplication equation is 2 x P = 14.* *I found the answer by skip counting 2, 4, 6, 8, 10, 12, 14.* *14 is the 7th number so there are 7 pairs.* *Student B* *My division equation is 14 ÷ 2 = P.* *My multiplication equation is 2 x P = 14.* *I found the answer by drawing groups of 2 circles until I had 14 circles.* 

Return to Standards

Solve two-step problems.
NC.3.OA.8 Solve two-step word problems using addition, subtraction, and multiplication, representing problems using equations with a symbol for the unknown number.

Clarification	**Checking for Understanding**
This standard calls for students to explore, represent and solve two-step word problems that include addition, subtraction, and multiplication. The size of the numbers should be limited to related 3rd grade standards. Specifically, tasks that include addition and subtraction should include numbers within 1,000 (NC.3.NBT.2), Tasks including multiplication are limited to either single-digit factors (NC.3.OA.1, NC.3.OA.7) or multiples of 10 between 10 and 90 (NC.3.NBT.3). This standard calls for students to represent problems using equations with a letter to represent unknown quantities. Students should be able to write and solve an equation when given a context. Students are expected to work with equations in which unknowns are in all positions.	Tasks that include only addition and subtraction The bagel shop has 79 plain bagels, 87 cinnamon raisin bagels and some blueberry bagels. There are a total of 241 bagels. Write an equation to find the total number of blueberry bagels. Show your work and find the answer. *Possible answers:* *Possible equations- 79 + 87 + B = 241 OR B = 241 - 79 - 87* *I used expanded form to subtract 79 from 241 to get 162. And then I subtracted 87 from 162 to get 75.* *130 11 — 100 + ~~140~~ 11 — ~~200~~ + ~~40~~ + ~~1~~ — -70 - 9 — 100 + 60 + 2 = 162* *150 12 — ~~160~~ 12 — ~~100~~ + ~~60~~ + ~~2~~ — -80 - 7 — 70 + 5 = 75* Tasks that include multiplication Mike runs 3 miles a day. His goal is to run 50 miles. After 7 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution. *Possible answers:* *Student A* *7 x 3 + M = 50* *I solved the problem by first multiplying 7 and 3 to get 21. I then subtracted 21 from 50.* *50 - 20 = 30, 30 - 1 = 29.* *Student B* *I used a beginning middle and end chart.* *Mike had already run 3 miles per day for 7 days. His goal was to end with 50 miles. We need to find the middle.* | *Beginning* | *Middle* | *End* | | *7 x 3* | *÷ M* | *= 50* | *I know 7 times 3 is 21. Then I subtracted 21 from 50.* *50 - 20 - 1 is 50 - 20 = 30. 30 - 1 =29 so Mike needs to run 29 more miles.* Mrs. Rojas' class is trying to earn $205 for the local animal shelter. They clean the lunchroom and earn $20 per week for 7 weeks. • How much money do they still need to raise? • Explain your solution using pictures, numbers, or words. • Write an equation that includes all of the numbers and a variable for the amount of money that they still need to earn.

Explore patterns of numbers.
NC.3.OA.9 Interpret patterns of multiplication on a hundreds board and/or multiplication table.

Clarification

This standard calls for students to examine multiplication patterns using the visuals of a hundreds board and a multiplication table. This standard is foundational as it expects students to have opportunities to explore, recognize and explain patterns in mathematics. This work contributes to students' process of making generalizations about patterns, which is a foundational concept in algebraic thinking.

While working on this standard as well as skip counting strategies in NC.3.OA.1, students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction to investigate multiplication and division patterns. Students investigate multiplication tables in search of patterns and explain why these patterns make sense mathematically.

- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Checking for Understanding

What do you notice about the shaded numbers in the multiplication table?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Possible response:
When one changes the order of the factors, they will still get the same product, such as 6 x 5 = 30 and 5 x 6 = 30.

Select which statements are true for the highlighted numbers on the hundreds chart below. Explain why they are true or are not true.

Every number is a multiple of 3.
Every number is a multiple of 6.
Every number is even.
Every other number is even.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Answers:

Every number is a multiple of 3.- True. All the numbers are multiples of 3.
Every number is a multiple of 6.- False. Every other number is a multiple of 6 but every number is not a multiple of 6.
Every number is even. False. 3, 9, 15, and 21 are not even.
Every other number is even. True. 6, 12, 18, 24 are even.

Return to Standards

Use place value to add and subtract.

NC.3.NBT.2 Add and subtract whole numbers up to and including 1,000.

- Use estimation strategies to assess reasonableness of answers.
- Model and explain how the relationship between addition and subtraction can be applied to solve addition and subtraction problems.
- Use expanded form to decompose numbers and then find sums and differences.

Clarification

In this standard, students build on work in previous grades regarding strategies based on place value, the properties of operations, and relating addition to subtraction. Students should be able to use the expanded form of a number to calculate sums and differences. Students explain their thinking and show their work and verify that their answer is reasonable.

The table below provides examples of the strategies described in this standard.

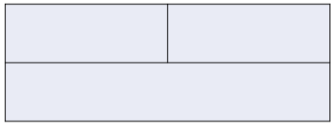
Strategy	Examples
Strategies based on place value (extended from Second Grade)	• Students use base ten blocks or draw pictures of base ten blocks to solve problems. • Students add or subtract in parts where they add hundreds, tens, and ones separately. This could be shown on a number line, a hundreds board (hundreds chart), in expanded form, or as equations. • Students add or subtract in parts where they add or subtract to land on a friendly or landmark number such as a multiple of 10 or 100. They also add or subtract the hundreds, tens, and ones separately.
Expanded form (new in Third Grade)	• A strategy based on place value where students decompose numbers by place (hundreds, tens, and ones) and then add and subtract each place one at a time. This is the strategy that most aligns with the U.S. standard algorithm which is introduced in Grade 4. 323 - 176 = ___ *110* *200 ~~120~~ 13* *~~300~~ + ~~20~~ + ~~3~~* *100 - 70 - 6* *100 + 40 + 7 = 147*

Checking for Understanding

One-step problem

At the store there are 323 pop-its. The number of pop-its is 176 more than the number of super bouncy balls.

Fill in the part-part-whole mat with the words pop-its, balls, and difference.



Write an equation to find the number of super bouncy balls.
Estimate the number of super bouncy balls.
Show your work and find the number of super bouncy balls.

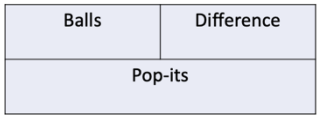
Possible Responses:

Student A

My equation is 323 - 176 = B

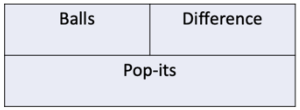
I used expanded form to solve the task.

110
200 ~~120~~ 13
~~300~~ + ~~20~~ + ~~3~~
100 - 70 - 6
100 + 40 + 7 = 147



Student B

Since the number of Pop-its is larger than the number of balls I need to add balls and the difference to get the number of pop-its.



My equation is 323 = B + 176

I estimated that 176 is close to 180 and 323 is close to 320 so my estimate is 320 - 180. I added up.
180 + 20 = 200
200 + 100 = 300
300 + 20 = 320. My estimate is 100 + 20 + 20 = 140.
The answer is 176 + __ = 323.
176 + 4 = 180
180 + 10 = 190
190 + 10 = 200
200 + 100 = 300
300 + 20 = 320
320 + 3 = 323.
My answer is 100 + 20 + 10 + 10 + 4 + 3 = 327

Use place value to add and subtract.

NC.3.NBT.2 Add and subtract whole numbers up to and including 1,000.

- Use estimation strategies to assess reasonableness of answers.
- Model and explain how the relationship between addition and subtraction can be applied to solve addition and subtraction problems.
- Use expanded form to decompose numbers and then find sums and differences.

Clarification

Properties of operations (extended from Second Grade)	• In third grade the commutative property of addition is the primary focus. Students change the order of the addends when adding multiple addends together OR when they have decomposed addends into tens and ones or decomposed addends into smaller numbers, they change the order of the addends. For example: 374 + 438 = __ 300 + 70 + 4 + 400 + 30 + 8. The student would decompose 8 into 6 and 2 so they can make a 10. 300 + 70 + 4 + 400 + 30 + 6 + 2 The commutative order of addition lets students rearrange the order of the addends. 300 + 400 + 70 + 30 + 4 + 6 + 2 700 + 100 + 10 + 2 = 812
Relationship between addition and subtraction (extended from Second Grade)	• Students rewrite a subtraction problem as an addition problem. For example, 612 - 328 = __ would be rewritten as 328 + __ = 612 OR solved by starting at 328 and adding in parts until they reach 612. When students add or subtract in second grade if they apply the relationship between addition and subtraction, they are expected to use strategies based on place value and/or properties of operations to find the answer.

Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties.

Checking for Understanding

Two-step problem (NC.3.NBT.2 and NC.3.OA.8)

There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground? The rest of the students on the playground are third graders. There are a total 601 students.

- Write an equation with a letter as the unknown to help find the number of third graders.
- Estimate the number of third graders.
- What is the total number of third graders on the playground?

Possible responses:

Possible correct equations: 178 + 225 + T = 601 (or 601 - 178 - 125 = T)
For my estimate, 178 is close to 200 and 225 is close to 200, and 601 is close to 600.
So, 200 plus 200 plus the third graders equals 600. There are about 200 third graders.

Student A
First I added 178 and 225 together by place value.

100 + 200 = 300
70 + 20 = 90
8 + 5 = 13
300 + 90 + 13 = 403 students

Then I subtracted 403 from 601 by place value.
600 – 400 = 200
I can't subtract 1 minus 3 so I traded 200 is 100 and 10 tens which is also 100 and 9 tens and 10 ones.
11 – 3 is 8 so I have 100, 9 tens and 8 ones which is 100+90+8 = 198.

Student B
I used expanded form to subtract.
First 601 – 178
90 11
500 ~~100~~
~~600~~ ~~+0~~ +1
-100 -70 -8
400 +20 +3

Now 423 – 225
110
300. ~~120~~ 13
~~400~~ ~~+20~~ ~~+3~~
-200 -20 -5
100 + 90 + 8

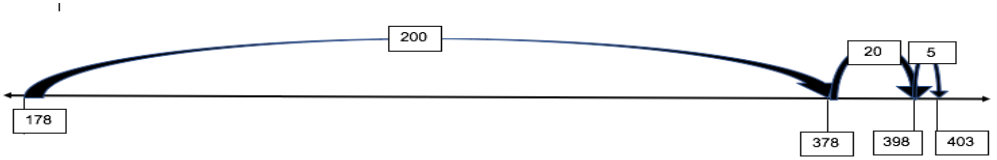
100 + 90 + 8 = 198

Student C
I added 178 and 225 first.
I know that 75 and 25 make 100 so 78 and 22 make 103.
100 + 200 = 300
75 + 28 = 103
300 + 103 = 403

I then added up to get to 601.
403 + 7 = 410
410 + 90 = 500
500 + 100 = 600
600 + 1 = 601

My answer is the sum of the numbers that I added
100 + 90 + 7 + 1 = 198

Student D
178 + 225 = ?
I split 225 into 200 + 20 + 5 and added each of those to 178.
178 + 200 = 378
378 + 20 = 398
398 + 5 = 403



Use place value to add and subtract.

NC.3.NBT.2 Add and subtract whole numbers up to and including 1,000.

- Use estimation strategies to assess reasonableness of answers.
- Model and explain how the relationship between addition and subtraction can be applied to solve addition and subtraction problems.
- Use expanded form to decompose numbers and then find sums and differences.

Clarification	Checking for Understanding
Estimation strategies include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations. For this standard, estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end, adjusting the estimate by taking into account the remaining amounts), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), Students should explore rounding using number lines and similar strategies. Rote memorization of rounding rules without conceptual understanding is not the expectation. The standard algorithm of carrying or borrowing is neither an expectation nor a focus in Third Grade. Students develop and use strategies for addition and subtraction in Grades K-3.	

Return to Standards

Generalize place value understanding for multi-digit numbers.
NC.3.NBT.3 Use concrete and pictorial models, based on place value and the properties of operations, to find the product of a one-digit whole number by a multiple of 10 in the range 10–90.

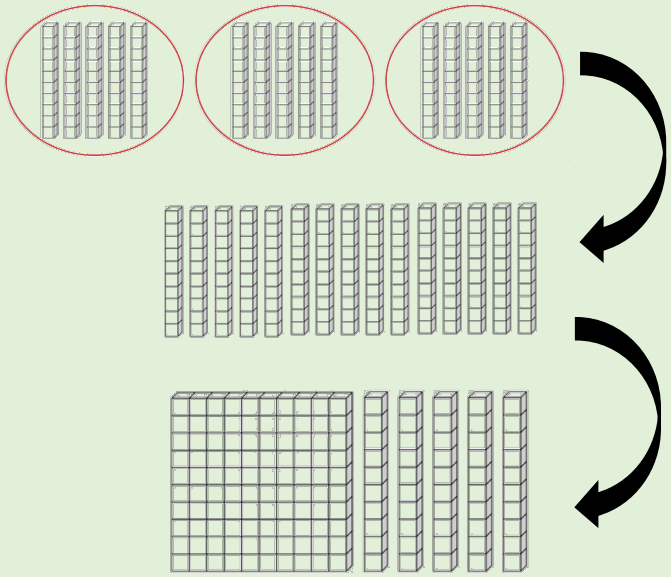
Clarification

In this standard, students extend on their work from multiplication by applying their understanding in tasks that involve the multiplication of a one-digit whole number by a multiple of 10 between 10 and 90. of place value.

The standard a focus on concrete and pictorial models (pictures and drawings of base ten or place value blocks). By using concrete and pictorial representations students extend the work from second grade when they explore and develop an understanding that 10 tens can be grouped together to make 100.

Using the properties of operations (commutative, associative, and decomposing a factor) and place value, students are able to explain their reasoning.

For example:
The product 3 × 50 can be represented as 3 groups of 5 tens, which is 15 tens, which is 150.



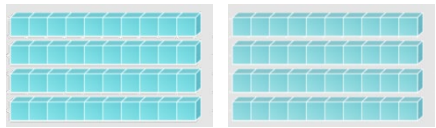
This standard can be integrated into NC.3.OA.8 in ways where students would solve a 2-step problem that involves the multiplication of a 1-digit number and a multiple of 10 from 10 up to 90.

Checking for Understanding

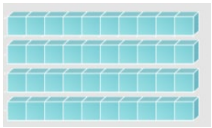
Serenity thinks that 40 x 3 is the same as 3 groups of 4 tens.
Malcolm thinks that 40 x 3 is the same as 4 x 3 x 10.
Which one is correct? Use base ten blocks to support your answer.

Possible answer:

Serenity is correct. 40 x 3 = 120. The picture shows 3 groups of 4 tens which is 3 groups of 40.



Malcolm is also correct. Using the picture, we can rearrange his factors into 3 x 4 x 10. The picture shows 3 groups with 4 groups of 10 in each group which is 3 x 4 x 10 which is 120.



Max is trying to decide if he should go to Fast Foods, Green Groceries, or Super Store to buy biscuits for the school picnic.

For $25, Max can buy:

- 60 five-packs of biscuits from Fast Foods.
- 30 six-packs of biscuits from Green Groceries.
- 40 eight-packs of biscuits from Super Store.

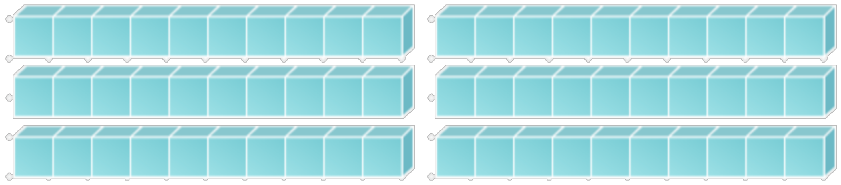
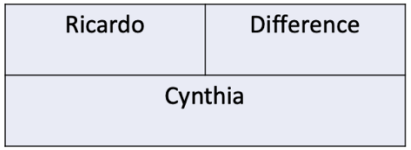
Where should Max go to buy biscuits? Use pictures, numbers, words, or equations to explain your reasoning.

Possible Responses:

Student A
60 x 5 is 5 groups of 6 tens which is 30 tens.
30 x 6 is 6 groups of 3 tens which is 18 tens
40 x 8 is 8 groups of 4 tens which is 32 tens.
Max can buy the most from the Super Store which had 32 tens or 320 biscuits.

Student B
60 x 5 = 6 x 5 x 10 = 300
30 x 6 = 3 x 6 x 10 = 180
40 x 8 = 4 x 8 x 10 = 320
Max can buy the most from the Super Store.

Generalize place value understanding for multi-digit numbers.
NC.3.NBT.3 Use concrete and pictorial models, based on place value and the properties of operations, to find the product of a one-digit whole number by a multiple of 10 in the range 10–90.

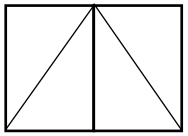
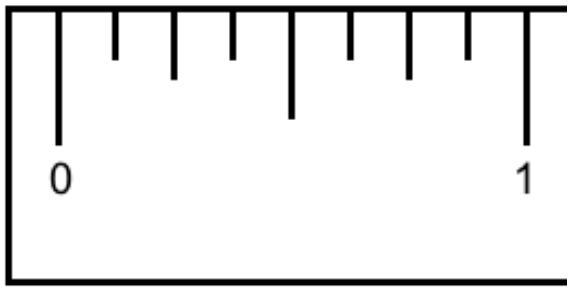
Clarification	Checking for Understanding
	NC.3.NBT.3 integrated with NC.3.OA.8 Cynthia has 2 packs of pencils and there are 30 pencils in a pack. Cynthia has 19 more than Ricardo. How many pencils do they both have? *Possible response:* *I used base ten blocks to show Cynthia's pencils. She has 60.*   *Since Cynthia has 19 more than Ricardo that means that Ricardo has 19 fewer than Cynthia.* *Ricardo + Difference = Cynthia or Cynthia - Difference = Ricardo.* *I subtracted 19 from 60.* *50 10* *~~60~~ + ~~0~~* *-10 - 9* *40 + 1 = 41. Ricardo has 41.* *Total = Cynthia + Ricardo* *Total = 60 + 41 =* *60 + 40 + 1 = 101*

Return to Standards

Understand fractions as numbers.

NC.3.NF.1 Interpret unit fractions with denominators of 2, 3, 4, 6, and 8 as quantities formed when a whole is partitioned into equal parts;

- Explain that a unit fraction is one of those parts.
- Represent and identify unit fractions using area and length models.

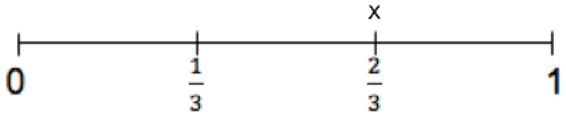
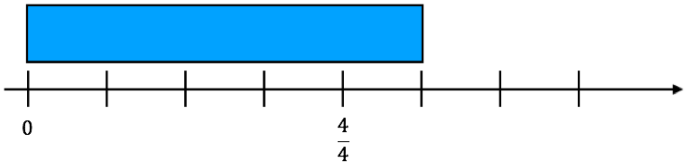
Clarification	Checking for Understanding
In this standard, students are expected to explain that a unit fraction represents one part of an area or length model of a whole that has been equally partitioned into 2, 3, 4, 6, or 8 parts. Area models may include rectangles, circles, or other 2-dimensional objects that can be partitioned into equal sized pieces. The most common length model is a number line. A unit fraction is a term that describes the size of 1 fractional piece in a whole. Students' work with NC.3.NF.1 related to unit fractions is foundational to NC.3.NF.2. Research suggests that it is developmentally appropriate to integrate NC.3.NF.1 and NC.3.NF.2 as long as there are explicit conversations about how a unit fraction (e.g, $\frac{1}{4}$) is the building block for other fractions with the same denominator (e.g., $\frac{3}{4}$) since fractions are made up of unit fractions (Empson & Levi, 2011). Student's experiences with this standard should extend their work in second grade when they developed an understanding that half of a half is a fourth. In Grade 3 students are expected to reason that fourths can be created by partitioning a half into two equal parts and that eighths can be created by partitioning a fourth into equal parts. Likewise, the relationship that a sixth can be created by partitioning pieces that are thirds into two equal parts should be explored and discussed. **For example:** $\frac{1}{3}$ is the unit fraction that identifies a whole being divided into 3 equal pieces. Just as there are 3 one-inch units in the length of 3 inches, there are 3 units of $\frac{1}{3}$ in the fraction $\frac{3}{3}$.	Tameka has a piece of paper like the one shown below. She colors in one of the triangles. How much of the paper has she colored?  Label the first section of the ruler, between 0 and 1 inch, to show how it is partitioned into 8 parts. Partition the section between 0 and 1 into fourths. Partition the section between 0 and 1 into halves. What did you notice about the size of the pieces each time you partitioned the ruler? 

Return to Standards

Understand fractions as numbers.

NC.3.NF.2 Interpret fractions with denominators of 2, 3, 4, 6, and 8 using area and length models.

- Using an area model, explain that the numerator of a fraction represents the number of equal parts of the unit fraction.
- Using a number line, explain that the numerator of a fraction represents the number of lengths of the unit fraction from 0.

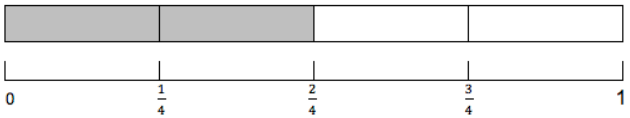
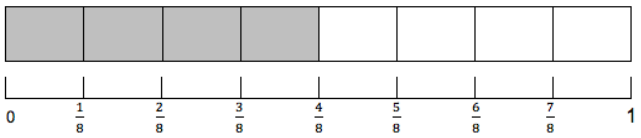
Clarification	**Checking for Understanding**
While working on NC.3.NF.2 students build off the work in NC.3.NF.1 to represent fractions with area and length models. Students are also expected to explain that fractions are composed of multiple iterations of the same unit fraction. Students' work with NC.3.NF.1 related to unit fractions is foundational to NC.3.NF.2. Research suggests that it is developmentally appropriate to integrate NC.3.NF.1 and NC.3.NF.2 as long as there are explicit conversations about how a unit fraction (e.g, $\frac{1}{4}$) is the building block for other fractions with the same denominator (e.g., $\frac{3}{4}$) since fractions are made up of unit fractions (Empson & Levi, 2011). In this standard, Students only work with fractions less than 1 except when measuring the length of an object.	Mrs. Turner says to the class, "Last weekend I saw a garden. The garden had equal sized sections and the following flowers: $\frac{3}{8}$ of the garden had red roses, $\frac{1}{8}$ of the garden had purple tulips, and $\frac{4}{8}$ of the garden had yellow sunflowers. Draw a picture of the garden and label the different parts. Write an explanation how you determined how to label the sunflowers. Mike runs on a straight road for 1 mile. Mike stops $\frac{2}{3}$ of the way down the road to stretch. Draw a number line that shows where Mike stopped to stretch. Write an explanation about how you knew where to mark where Mike stopped to stretch on the number line. *Possible student response:*  3.NF.2 integrated with 3.MD.2 where fractions may go beyond 1 Brandi is making hair bows. Each hair bow takes $\frac{1}{4}$ of a yard of ribbon to make. This is what she bought at the store:  If she uses all the ribbon, how many $\frac{1}{4}$ long is the ribbon? Express the length of the ribbon as fraction.

Return to Standards

Understand fractions as numbers.

NC.3.NF.3 Represent equivalent fractions with area and length models by:

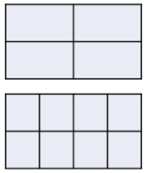
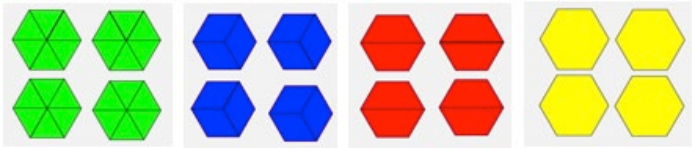
- Composing and decomposing fractions into equivalent fractions using related fractions: halves, fourths and eighths; thirds and sixths.
- Explaining that a fraction with the same numerator and denominator equals one whole.
- Expressing whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Clarification	**Checking for Understanding**
Students are expected to use area and length models to compose and decompose fractions into equivalent fractions using related fractions: halves, fourths, eighths, thirds, and sixths. Related fractions are fractions in which one denominator is a multiple of the others; thirds and sixths are related fractions, while fourths and sixths are not related fractions. The concept of equivalent fractions naturally is seen as students explore fractions while folding paper and drawing fraction models with NC.3.NF.2. For example, as students make the fraction $\frac{2}{4}$ some students will see that it is equivalent to the fraction $\frac{1}{2}$. <u>Fractions with the same numerator and denominator equal one whole:</u> NC.3.NF.3 also calls for students to explain that fractions with the same numerator and denominator equal one whole. Renaming fractions with the same numerator and denominator as one whole without a model is not sufficient for this standard. <u>Expressing whole numbers as fractions:</u> The standard also expects students to express whole numbers as fractions. This work is limited to whole numbers less than 4 (see example in Checking for Understanding). Expressing whole numbers as fractions lay the groundwork for seeing a fraction as a division problem, e.g., the fraction 4/2 represents 4 pieces that are a half each that equal 2 wholes. This standard is the building block for later work in Grade 5 where students divide a set of objects into a specific number of groups. Please note that the term “improper fraction” can cause developmental misconceptions. “This term can be a source of confusion as the word improper implies that this representation is not acceptable, which is not the case at all--in fact, it is often the preferred representation in algebra. Instead, try not to use the phrase and instead use “fraction” or “fraction greater than 1” (Van deWalle, Karp, Bay-Williams, 2019).	<u>Composing and decomposing fractions into equivalent fractions</u> Finlay and Peyton were talking about the fraction $\frac{1}{2}$ and equivalent fractions. Finlay drew a rectangle and a number line to show how many fourths were equal to one-half, while Peyton drew a rectangle and a number line to show how many eighths were equal to one-half. Draw pictures that match what Finlay and Peyton each drew. Write an explanation to support your pictures. *Possible student response:* Finlay *I shaded in half of my picture. I knew that if I split each half into 2 parts I would get fourths. I found that $\frac{2}{4} = \frac{1}{2}$. I then drew a number line to see if it worked there and it did.*  Peyton *I started by shading half the rectangle and marking $\frac{1}{2}$ on the number line. Then I divided each half into 4 equal sections to give me 8 sections in my rectangle and 8 sections of my number line. I found that when $\frac{4}{8}$ of the rectangle is shaded that is the same as half of the rectangle so $\frac{4}{8} = \frac{1}{2}$. On the number line when I jump by 8ths to $\frac{4}{8}$, $\frac{4}{8}$ is also at the same point as $\frac{1}{2}$.*  The Currituck Cakery is making rectangular cakes that are the same size but is cutting them into different pieces. The cakebaker, Catarina, cuts cakes into halves, fourths, and eighths. The customers are confused. Catarina explains to them that when you buy a cake that is cut into fourths the entire cake is still equal to a cake that is cut into eighths. Draw a picture of a cake cut into fourths. Draw a picture of a cake cut into eighths. Write a sentence and explain why both cakes are the same amount.

Understand fractions as numbers.

NC.3.NF.3 Represent equivalent fractions with area and length models by:

- Composing and decomposing fractions into equivalent fractions using related fractions: halves, fourths and eighths; thirds and sixths.
- Explaining that a fraction with the same numerator and denominator equals one whole.
- Expressing whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Clarification	Checking for Understanding
	Possible response: *I drew two rectangles that are the same size. I cut one into fourths and one into eighths.* *The cake cut into fourths is* $\frac{4}{4}$. and the cake cut into eighths is $\frac{8}{8}$. Both of those fractions are equal to 1 whole.  Expressing whole numbers as fractions Mrs. Floyd has a lot of hexagon shaped pies. She takes 4 pies and cuts each pie into sixths. She takes 4 pies and cuts each pie into halves. She takes 4 pies and cuts each pie into thirds. Then she takes 4 pies and leaves them whole. For each set of pies use pattern blocks to make a model how she cut the 4 pies. Then write a fraction equal to 4 to show the number of pieces compared to the size of each piece. *Possible response:*  *The fraction for pies cut into sixths would be 24 pieces where 1 whole is divided into sixths, so the fraction is* $\frac{24}{6}$ *= 4.* *The fraction for pies cut into thirds would be 12 pieces where 1 whole is divided into thirds, so the fraction is* $\frac{12}{3}$ *= 4.* *The fraction for pies cut into halves would be 8 pieces where 1 whole is divided into halves, so the fraction is* $\frac{8}{2}$ *= 4.* *The fraction for pies left whole would be 4 pieces where 1 whole is still 1 whole, so the fraction is* $\frac{4}{1}$ *= 4.*

Return to Standards

Understand fractions as numbers.

NC.3.NF.4 Compare two fractions with the same numerator or the same denominator by reasoning about their size, using area and length models, and using the >, <, and = symbols. Recognize that comparisons are valid only when the two fractions refer to the same whole with denominators: halves, fourths and eighths; thirds and sixths.

Clarification

This standard involves comparing fractions with or without area and length fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that $\frac{1}{3}$ of a cake is larger than $\frac{1}{4}$ of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.

In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, $\frac{1}{2}$ of a large pizza is a different amount than $\frac{1}{2}$ of a small pizza. Students should be given opportunities to discuss and reason about which $\frac{1}{2}$ is larger.

Students also see that for unit fractions, the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical) pieces to make the same whole, the pieces must be smaller. From this, students reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example, $\frac{2}{4} > \frac{2}{8}$, because $\frac{1}{8} < \frac{1}{4}$4, so 2 lengths of $\frac{1}{8}$ are less than 2 lengths of $\frac{1}{4}$.

The use of strategies such as cross multiplying and the butterfly procedure are not appropriate in elementary school since they do not make explicit connections between comparing fractions and the visual fraction models. Students' work should be focused on using manipulatives or drawings to represent fractions or reasoning about their size.

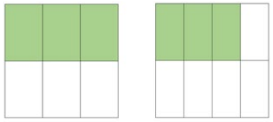
While this standard says that two fractions are compared, assessment items may include up to 4 fractions that students will be expected to order in ascending (least to greatest) or descending (greatest to least) order based on the understanding that ordering those numbers includes comparing two fractions at a time.

Checking for Understanding

Luke and Tyrone each buy a medium pizza. Luke has his pizza cut into 8 pieces while Tyrone has his pizza cut into 6 pieces. If they each eat 3 pieces, who ate more? Draw a picture and write an explanation about how you know you are correct.

Possible response:

I drew rectangles that were the same size. Luke ate less than half of his pizza, while Tyrone at exactly half of his pizza. Tyrone ate more than Luke.



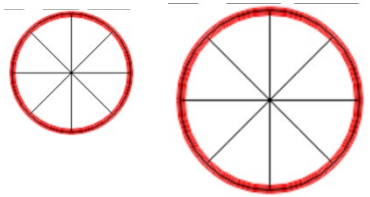
Tyrone Luke

Harriet and Monique are each eating a piece of licorice. Harriet eats $\frac{4}{6}$ of her piece while Monique eats $\frac{5}{6}$ of her piece. Who ate less? Draw a picture and write an explanation about how you determined who ate a smaller amount?

Possible response:

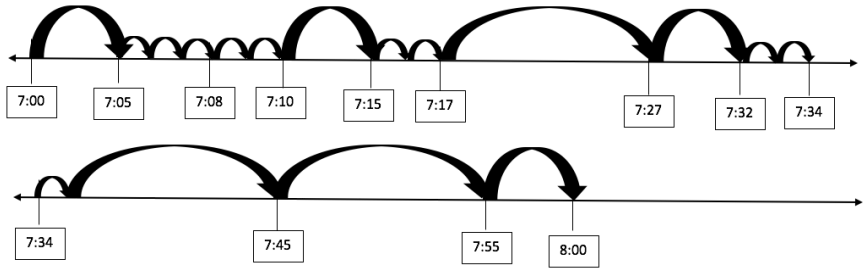
Mr. Tobias bought a small cake and Mrs. Kalvicky bought a medium cake. They each ate $\frac{1}{8}$ of their cake. Did they eat the same amount? Draw a picture and write an explanation about how you determined your answer.

Possible response:



Even though both cakes are cut into eighths, Mrs. Kalvicky has a larger cake so $\frac{1}{8}$ of her cake is larger than $\frac{1}{8}$of the cake that Mr. Tobias has. I can't compare eighths when the wholes are different sizes.

Return to Standards

Solve problems involving measurement. **NC.3.MD.1** Tell and write time to the nearest minute. Solve word problems involving addition and subtraction of time intervals within the same hour.	
Clarification	**Checking for Understanding**
In this standard, students will be able to tell time to the nearest minute, and determine elapsed time, including solving word problems. The expectation for third grade, is that students determine elapsed time within the same hour. Students are not expected to determine elapsed time over the hour. Specifically, problems such as finding the time that is 45 minutes before 3:30 p.m. are not appropriate since it would require students to cross over the hour. When solving word problems involving time intervals, students should use strategies for addition and subtraction to find an end time, amount of time passed, or a start time within an hour. Students should use tools such as clocks, timelines, and tables to solve problems.	At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem. *Possible response:* 

Return to Standards

Solve problems involving measurement.

NC.3.MD.2 Solve problems involving customary measurement.

- Estimate and measure lengths in customary units to the quarter-inch and half-inch, and feet and yards to the whole unit.
- Estimate and measure capacity and weight in customary units to a whole number: cups, pints, quarts, gallons, ounces, and pounds.
- Add, subtract, multiply, or divide to solve one-step word problems involving whole number measurements of length, weight, and capacity in the same customary units.

Clarification

In this standard, students estimate and measure length, capacity and weight using customary units. Students need foundational experiences measuring real-world objects in order to develop a basic understanding of the size and weight of customary units and apply this understanding when estimating and measuring.

Students are not expected to convert between units. The focus of measurement work in Grade 3 focuses on measuring, estimating, and using benchmarks to reason about the lengths, weights, and capacities of objects and the amount of liquid in containers.

<u>Measurement of lengths to the quarter-inch and half-inch</u>

The first bullet of this standard provides a context for students to explore the idea of mixed numbers (e.g., $2\frac{1}{2}$, $1\frac{3}{4}$) when they measure the length of objects to the quarter-inch and half-inch. Since this is the only context in Grade 3 where students work with mixed numbers, students should have hands-on experiences to measure lengths that are greater than 1 inch and have a fractional part to them. The goal of this part of the standard is to provide students with experiences to make sense of the idea that an object that has a length of $2\frac{1}{2}$ inches is longer than 2 inches and approximately halfway between 2 and the next whole number which is 3.

Word problems related to this standard are limited to one-step problems where all measurements include the same unit. The number range for these tasks should match the number size described in the OA and NBT standards.

Checking for Understanding

Use a ruler and measure 5 objects in the classroom in inches. If objects have measurements that are not whole inches, measure to the half or quarter inch.

Possible response:

Red crayon: 4 and $\frac{1}{4}$ inches
Length of math book: 11 inches
Piece of journal paper: 10 and $\frac{1}{2}$ inches

Estimate the following:
The amount of water in a bathtub

400 cups 400 quarts 400 gallons

For each container estimate or measure its capacity in cups, pints, quarts, or gallons.

Container	Estimate	Actual Measurement
3 large milk containers at the store		
Trash can		
Small milk container in the cafeteria		

---

20 cups of water come out of the faucet into a sink each minute. How much water is in the sink after 6 minutes?

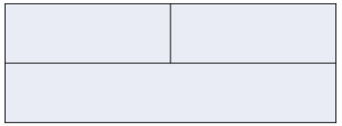
Possible Response:

20 is 2 tens. So, if 2 tens come out each minute then 12 tens or 120 will come out in 6 minutes.

Solve problems involving measurement.

NC.3.MD.2 Solve problems involving customary measurement.

- Estimate and measure lengths in customary units to the quarter-inch and half-inch, and feet and yards to the whole unit.
- Estimate and measure capacity and weight in customary units to a whole number: cups, pints, quarts, gallons, ounces, and pounds.
- Add, subtract, multiply, or divide to solve one-step word problems involving whole number measurements of length, weight, and capacity in the same customary units.

Clarification	Checking for Understanding
	Before being shipped to stores, the crayons in the factory are weighed. The red crayons have a mass of 504 ounces. The mass of the red crayons is 129 ounces more than the mass of black crayons. Fill in the diagram with the words *red, black,* and *difference*. Write an equation with the numbers in the problem and use a letter as the unknown to represent the mass of the black crayons. Show your work and find the mass of the black crayons?  *Possible response:* *B + 129 = 504 OR* *504 - 129 = B*  *I used expanded form to find the answer.* *90* *400 ~~100~~ 14* *~~500~~ + ~~0~~ + ~~4~~* *-100 - 20 - 9* *300 + 70 + 5 = 375. The mass of the black crayons is 375 ounces.*

Return to Standards

Represent and interpret data.

NC.3.MD.3 Represent and interpret scaled picture and bar graphs:

- Collect data by asking a question that yields data in up to four categories.
- Make a representation of data and interpret data in a frequency table, scaled picture graph, and/or scaled bar graph with axes provided.
- Solve one and two-step "how many more" and "how many less" problems using information from these graphs

Clarification

In this standard, students will interact with data. Students should have experiences with posing questions, collecting data, analyzing data (including the creation of graphs), and interpreting data.

In Grade 3 students work with data focuses exclusively on questions that yield categorical data, where the question includes up to 4 categories or choices. Representations of categorical data in Grade 3 are limited to frequency tables, scaled picture graphs, and scaled bar graphs. When given data, students are expected to create an appropriate graph correctly. Graphs should include a title, categories, category label, key, and data. Once graphs are created, students should be able to solve simple one and two-step problems using the information in the graphs.

Scaled picture graphs have pictures that represent more than 1 data point. Scaled bar graphs have a scale on the y-axis in which the labels do not include every number. Both scaled types of graphs could include data that includes half of an object on picture graphs or bar graphs in which a bar is in between labels. The work with scaled graphs is a natural integration with the work done in multiplication (NC.3.OA.1) and division (NC.3.OA.2).

The last bullet asks students to solve one-step and two-step word problems based on data that is in scaled picture graphs and scaled bar graphs. These word problems are limited to only addition and subtraction to answer "how many more" and "how many less/fewer" questions.

Checking for Understanding

Maria wanted to know what flavor of juice the people in her class like the most. What question could Maria ask? How could she collect the data?

Possible response:

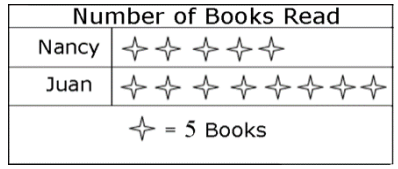
Maria can ask each person in her class, "Which of these four flavors is your favorite type of juice?" Maria can keep track of the votes on a frequency table.

Flavor	People
Grape	𝍸
Cherry	𝍸 𝍸 |
Apple	𝍸 ||
Orange	||

Flavor	People
Grape	5
Cherry	11
Apple	7
Orange	2

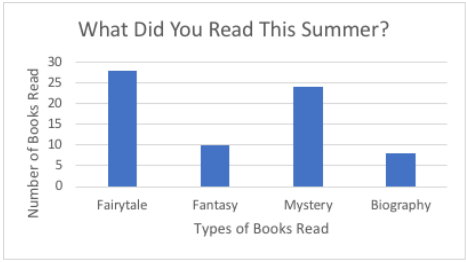
Nancy and Juan read the following number of books during the summer.

- How many books did they read together?
- How many more books did Juan read compared to Nancy?
- Sarah read more books than Nancy but less books than Juan. How many books could Sarah have read?



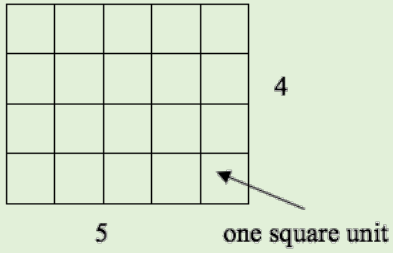
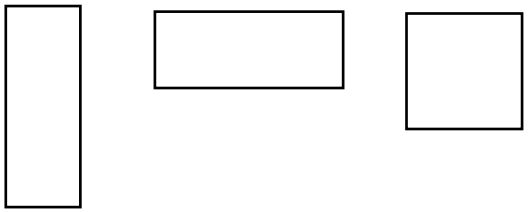
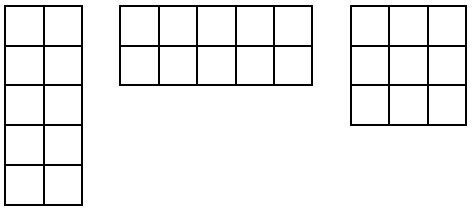
As a class we are going to design a survey to collect data from all of the third grade students in the school about the types of books they read this summer. You will take the data from a table and make it into a scaled bar graph. Then write and solve two math problems that compare the values in your graph.

Possible response:



Return to Standards

Understand the concept of area.
NC.3.MD.5 Find the area of a rectangle with whole-number side lengths by tiling without gaps or overlaps and counting unit squares.

Clarification	Checking for Understanding
This standard calls for students to explore the concept of area as the covering of a region with unit squares. Students should understand that a unit square is a square with side length 1 unit and has one square unit of area and should be able to make connections between the number of squares it takes to cover an area and the dimensions of the rectangle. While working with this standard and multiplication (NC.3.OA.1) students should have ample hands-on opportunities with square tiles to build arrays and determine the array by counting the unit squares, using repeated addition, and then eventually developing an understanding that the area can be found by multiplying the length and the width of a rectangle (NC.3.MD.7). Students should be able to count the square units to find the area. Units could include metric, customary, or non-standard square units. **For example:** In the figure below, there are 20 square units. Each square unit is a square with the side length of 1 unit. The rectangle is 5 units long and 4 units wide.  	Use the square tiles to cover each shape. Then, find the area of the shapes below. Which rectangle is the largest?  *Possible response:*  *The left and middle rectangles are 10 square units. The right rectangle is 9 square units. The left and middle rectangles are the largest.*

Return to Standards

Understand the concept of area.

NC.3.MD.7 Relate area to the operations of multiplication and addition.

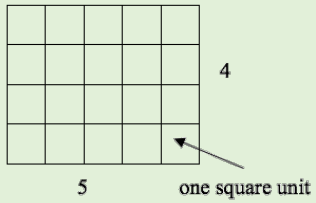
- Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths.
- Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving problems and represent whole-number products as rectangular areas in mathematical reasoning.
- Use tiles and/or arrays to illustrate and explain that the area of a rectangle can be found by partitioning it into two smaller rectangles, and that the area of the large rectangle is the sum of the two smaller rectangles.

Clarification

In this standard, extend their understanding developed with covering rectangles (NC.3.MD.5) to move from counting squares to repeated addition to multiplying the length and the width of a rectangle in order to find the area.

Students are expected to explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior. Students who multiply the dimensions to find the area without providing a clear reason why multiplying works have not met the expectation for this standard.

For example: In this rectangle, there are 4 rows of 5 units squares, or 5 columns of 4 unit squares. Students should tile rectangle to find that there are 20 square units, then multiply the side lengths to show it is the same.



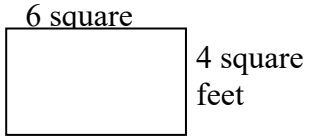
4 x 5 = 20
5 x 4 = 20

Students also are expected to decompose a rectangle into two smaller rectangles in order to find the area of the smaller rectangles and the larger rectangle. This work overlaps the idea of decomposing a factor (NC.3.OA.1, NC.3.OA.2). The dimensions of the smaller rectangles should be 10 or less.

This standard also addresses using multiplication to determine area while solving word problems (NC.3.OA.3). Students are expected to determine the possible dimensions of a rectangle when the area is given.

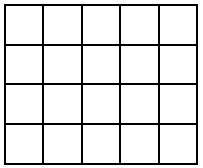
Checking for Understanding

Sam wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?



You have a rectangle that is 5 inches by 4 inches. Explain why multiplying the dimensions of the rectangle is an appropriate strategy to find the area.

Possible responses:

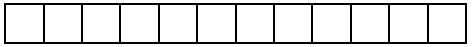


Student A: I know that I have 5 columns and 4 rows. I added the number 5, 4 times. 5+5+5+5 = 20. That is the same as 5x4 which is 20. Since I got the same answer when I added 5+5+5+5 and 5x4

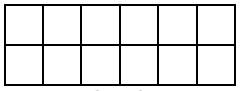
Student B: I counted all of the squares and there were 20. I know that 5x4 is 20 so I get the correct answer.

The area of a rectangular playpen for a guinea pig is 12 square yards. What are the possible dimensions of the playpen?

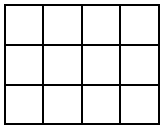
Possible response:



1 x 12



2 x 6

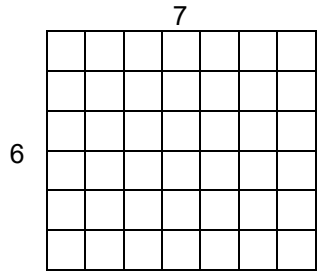
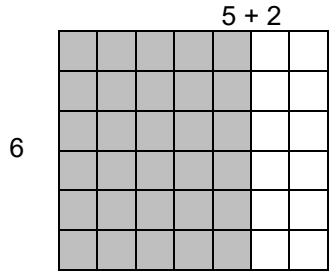
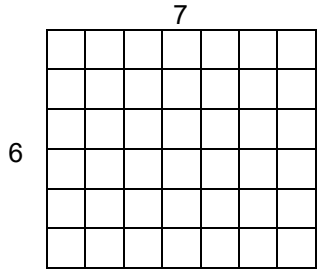
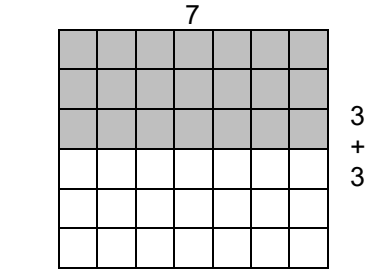


3 x 4

Understand the concept of area.

NC.3.MD.7 Relate area to the operations of multiplication and addition.

- Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths.
- Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving problems and represent whole-number products as rectangular areas in mathematical reasoning.
- Use tiles and/or arrays to illustrate and explain that the area of a rectangle can be found by partitioning it into two smaller rectangles, and that the area of the large rectangle is the sum of the two smaller rectangles.

Clarification	Checking for Understanding
	You buy a rectangular carton of candy that has 7 columns and 6 rows. Find two different ways to split the rectangle.   7 x 6 = (5 x 6) + (2 x 6)   7 x 6 = (7 x 3) + (7 x 3)

Return to Standards

Understand the concept of perimeter.
NC.3.MD.8 Solve problems involving perimeters of polygons, including finding the perimeter given the side lengths, and finding an unknown side length.

Clarification	Checking for Understanding
In this standard, students develop an understanding of the concept of perimeter as the distance around a shape. In Grade 3 students are expected to find the perimeter of polygons as well as use addition and subtraction to find an unknown side length when given the perimeter and the lengths of some of its sides. The word *regular* is included in this standard and means that each side is the same length. Specifically, a regular hexagon has 6 sides that are the same length. While determining the perimeter of polygons students are expected to integrate their knowledge of shapes (NC.3.G.1), multiplication (NC.3.OA.1), and division (NC.3.OA.2) to find the perimeter of a regular polygon when given the side length or find the length of each side when given the perimeter.	You have 24 feet of fencing. What are the possible dimensions you can have for a rectangular fenced in area? *Possible response:* *The tallest and skinniest rectangle is 11 feet tall and 1 foot wide so it is 11 by 1. I then realized I could make shorter rectangles with the same perimeter. When I made one that was 10 tall the width was 2 so the dimensions were 10 by 2. I continued to do that so I had rectangles that were 9 by 3, 8 by 4, a 7 by 5, and 6 by 6.* A regular hexagon has a perimeter of 54 inches. What is the length of each side? *Possible response:* *A hexagon has 6 sides. Since it is a regular shape that means that each side is the same length. I need to find 6 x __ = 54.* *6 x 5 = 30. Then I skip counted by 6: 36, 42, 48, 54* *6 x 9 =54 so each side is 9 inches.*

Return to Standards

Reason with shapes and their attributes.

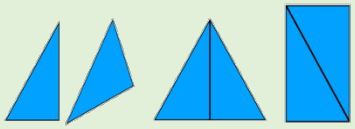
NC.3.G.1 Reason with two-dimensional shapes and their attributes.

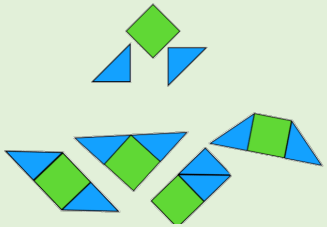
- Investigate, describe, and reason about composing triangles and quadrilaterals and decomposing quadrilaterals.
- Recognize and draw examples and non-examples of types of quadrilaterals including rhombuses, rectangles, squares, parallelograms, and trapezoids.

Clarification

In this standard, students explore triangles and quadrilaterals. Students move beyond identifying and classifying triangles and quadrilaterals to manipulating two or more shapes to create other triangles and quadrilaterals. Students should be able to describe the shapes they have composed using informal geometric terminology and understand the relationship between the components of the new shape.

For example:
Students can manipulate two right triangles to create another triangle. They can also manipulate the triangles to compose a rectangle.





Students can manipulate a square and two triangles to create a variety of triangles and quadrilaterals. Students should be able to describe the composite shapes using attributes of triangles and quadrilaterals.

Students examine the properties of quadrilaterals and determine whether or not a shape is a quadrilateral. Students understand that a quadrilateral must be a closed figure with four straight sides and four angles and should be able to describe the characteristics of quadrilaterals including details about the angles and the relationship between opposite sides. Students should able to sort geometric figures and identify squares, rectangles, rhombuses, parallelograms, and trapezoids as quadrilaterals.

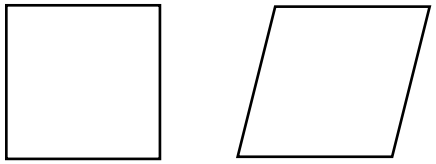
In Grade 3 right angles should be referred to as a square corner and the different types of angles (acute, right, obtuse, straight) should be left for Grade 4. The concept of *parallel* should be referred to as opposite sides moving in the same direction until Grade 4.

Note: North Carolina has adopted the exclusive definition for a trapezoid. A trapezoid is a quadrilateral with *exactly* one pair of parallel sides.

Checking for Understanding

Draw a picture of a square. Draw a picture of a rhombus.
How are they alike? How are they different?

Possible response:



A square and a rhombus both have 4 sides. All four sides are the same length. A square has four equal angles, and a rhombus does not. The opposite angles are equal.

Manny drew a quadrilateral that had 2 or more square corners.

- Could he have drawn a trapezoid? Draw a picture and explain why or why not?
- Could he have drawn a parallelogram? Draw a picture and explain why or why not?
- Could he have drawn a rectangle? Draw a picture and explain why or why not?
- Could he have drawn a square? Draw a picture and explain why or why not?

Possible Response:

Manny could have drawn a trapezoid if he has 2 square corners, and it looks like the shape that I drew.

I know that a parallelogram has opposite sides that move in the same diagram. A rectangle has 4 square corners, and the opposite sides move in the same direction. So if he drew a rectangle he also drew a parallelogram since rectangles are types of parallelograms.

I know that a rectangle has 4 square corners so he could have drawn a rectangle.

I know that a square has 4 square corners so he could have drawn a rectangle.

Return to Standards